Optimal Deployment of Drifting Acoustic Sensors:
Sensitivity of Lagrangian Coherent Structure Boundaries
to Model Uncertainty

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LONG-TERM GOALS

Our long-term goal is to provide a template for the launch locations of drifting Lagrangian measurement platforms that will maximize the amount of environmental information provided by a necessarily limited number of observational resources. By coupling the geometric analysis of fluid parcel movement provided by the identification of Lagrangian coherent structure boundaries with high resolution, Lagrangian data assimilating numerical ocean models, we ultimately seek the accurate prediction of particle trajectories in the ocean. The overall goal is develop a robust set Lagrangian analysis tools that provide synoptic information on the fate of drifting sensor packages.

OBJECTIVES

We seek to couple the information provided by knowledge of coherent structure boundaries defined by the manifolds emanating from hyperbolic trajectories with available Lagrangian data assimilation techniques to provide directed launch strategies for drifting sensor packages. The objectives of the first phase of this project include:

- Quantification of improvements in assimilation accuracy using directed launch procedures based on Lagrangian geometric analyses in ocean models assimilating drifter information.

- Development of metrics, based on the geometry of Lagrangian Coherent Structure boundaries, to assess and compare ocean model performance in the Lagrangian frame.

- Determination of the sensitivity of computed Lagrangian coherent structure boundaries to unavoidable errors and uncertainties in the underlying Eulerian fields given by numerical models and observations.

- Transition of available Lagrangian Coherent Structure identification techniques to the analysis of the latest, high resolution, data-assimilating state-of -the-art ocean models of Naval relevance.

APPROACH

Optimal deployment strategies for drifting acoustic arrays will rely heavily on precise knowledge of the location, in both time and space, of Lagrangian coherent structure boundaries computed by post-

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processing numerical Eulerian velocity data. A fundamental question concerning the operational use of such Lagrangian analyses is the sensitivity of the computed structures to inherent uncertainties in the model prediction of the Eulerian velocity field. Objective assessment of the level of Lagrangian uncertainty produced by current ocean models requires the development of metrics that will allow quantitative comparison of the extended Lagrangian boundary curves resulting from ensembles of model perturbations. Systematic analysis of the sensitivity of computed LCS boundaries to various levels of model uncertainty will determine the model precision and fidelity needed to reliably employ LCS boundary information in an operational observational experiment.

The approach of the research over the past year has been to examine, in detail, the application of Lagrangian analysis tools to the output of two particular numerical models. The first is CUPOM Gulf of Mexico model supplied by Lakshmi Kantha of the University of Colorado. This POM-based model assimilates sea-surface height and NOGAPS wind data and has been a test bed for assessing the applicability of geometric Lagrangian techniques, originally developed for simple analytic vector fields, to the output of reasonably high resolution ocean models. Working with Kantha and Denny Kirwan's group at the University of Delaware, we have investigated the role played by identifiable LCS boundaries in determining the ultimate fate of modeled Loop Current Rings as they enter the western basin of the Gulf of Mexico. By means of a number of Lagrangian metrics including maps of Finite Scale Lyapunov Exponents, two-particle dispersion statistics, and the computation of finite time manifolds extending from transient hyperbolic trajectories, we have shown that energetic Loop Current Rings in the model lose coherence rapidly in deep water far from the western shelf. Such is not the case in a number of non-data assimilating GOM models, including the CUPOM with identical resolution run without data assimilation. The suite of analyses provides a reproducible set of metrics for testing the hind-casts of other GOM models in the Lagrangian frame of reference.

The CUPOM data set has also been used to address the primary research goal of assessing and quantifying the sensitivity of computed LCS boundaries to model uncertainty. For the Gulf of Mexico, a body of previous work allows us to identify sufficiently strong hyperbolic trajectories in the model output and compute the corresponding manifold geometry. How robust such Lagrangian structures are to the details of the computation, such as the spatial and temporal resolution, specifics of the turbulence and forcing parameterizations, et cetera, is the subject of debate. As a proxy for examining the individual effects of changing such model parameterizations, and to avoid the prohibitive cost of re-running the ocean model for each case, we study the effects of increasingly coarse spatial smoothing operators on the CUPOM output by comparing the resulting LCS boundaries at each level of averaging.

A similar approach is taken using data produced by the Navy using a very high spatial resolution (dx ~ 1 km, dt ~ 1 hr) NCOM model of the Adriatic basin. This data, provided by Paul Martin of NRL, Stennis, is part of the NATO sponsored Dynamics of the Adriatic in Real Time, DART05, experimental program. This large-scale observational effort, originally planned for September 2005, aims to deploy a variety of monitoring resources, including a number of Lagragian platforms, concentrated in the vincinity of the Gargano peninsula in the Adriatic. The goals include synoptic mapping of the velocity and hydrography and testing of data assimilating models. This aspect of the project is joint work with Tamay Ozgokmen and Angelique Haza at the Rosentiel School of Marine and Atmospheric Sciences at the University of Miami.

The high resolution of the available model output in both time and space provides a means for testing the sensitivity of computed Lagrangian templates to both temporal and spatial smoothing procedures.

Also, the availability of a historical database of Adriatic drifter trajectories through Annalisa Griffa, RSMAS, will eventually allow for model-data inter-comparison studies in the Lagrangian frame.

Initially, we concentrate on the use of Finite-Scale Lyapunov Exponents (FSLE) as a proxies for the LCS boundaries formally given by computation of stable and unstable manifolds. In short, one examines the time required for particle pairs initially separated by some small distance, to diverge to a certain finite scale. The inverse of this time is, in the appropriate limits, a measure of the Lyapunov Exponent of the initial condition. The use of FSLEs in understanding and quantifying mixing is now well accepted (see for example D'Ovidio *et al.*). Direct comparison of FSLE structure to manifold computations has been done in Molcard, Poje & Ozgokmen, 2006.

WORK COMPLETED

A Lagrangian analysis of the fate of three well-defined Loop Current Rings was conducted in the context of Kantha's CUPOM Gulf of Mexico model. In collaboration with Lipphardt at the University of Delaware, an extensive software suite of MatLab tools was developed to easily examine a number of Lagrangian based metrics in the context of the numerical model output. Functionality includes efficiently calculating large numbers of model trajectories, tracking Eulerian saddle and elliptic points, constructing finite time manifolds, FSLE fields and interpolating model quantities such as temperature and salinity along numerical particle paths. These tools are readily portable to output of different ocean models. A manuscript addressing the Lagrangian analysis of the Loop Current Rings has been submitted to *Ocean Modeling*.

Analysis of Lagrangian Coherent Structure boundaries in terms of FSLE computations has begun for the DART05-06 region of the NCOM Adriatic model. Results on the sensitivity of structure boundaries to both temporal and spatial smoothing are presented below. The software suite previously developed under this funding for the automatic computation of hyperbolic trajectories, manifold generation and the generation of trajectory statistics has been adapted and modified to these model fields. Efficient and reliable software for computing FSLE fields and averages has been developed. This work is a collaborative effort with T. Ozgokmen and his post-doctoral researcher A. Haza. A manuscript (A. Haza, P. Martin, T. Ozgokmen, & A.C. Poje, Scale dependent Lagrangian structure in the Adriatic Sea: Model analysis) is currently in preparation for submission to *Ocean Modeling* this year.

For both data sets, fast and reliable software for computing spatial convolutions over smoothing operators was developed.

Collaborative research with Ozgokmen and Molcard on the use of Lagrangian coherent structure boundaries to develop directed launch strategies that enhance assimilation accuracy was completed in year 2 of the project, with a manuscript appearing in *Ocean Modeling* earlier this year.

RESULTS

Figure 1 shows the results of applying successively coarser spatial smoothing to the output of the CUPOM Gulf of Mexico model. Shown are contours of the meridional velocity component under the action of a Gaussian spatial filter. The quantity varied is the standard deviation of the Gaussian measured in grid-cell units, dx ~ 12 km. Application of the filter, even at the smallest window width,

produces considerable smoothing of the rather noisy raw model data. Smoothing at the larger window widths completely obscures a number of meso-scale features present in the raw data.

Figure 2 shows the sensitivity of computed LCS boundaries to the spatial filtering for two distinct flow features separated in time by 18 months. In each, we are interested in the LCS boundaries that lead to the rapid stretching, folding and eventual loss of coherence of energetic Loop Current Rings as they enter the Western basin of the Gulf of Mexico. The top panel of Figure 2 depicts the splitting of eddy *Fourchon* along the unstable, out-flowing manifold of a hyperbolic trajectory located near 268W, 24.25N. The bottom panel depicts the LCS boundary that prevents further westward propagation of eddy *Millennium*. In each, the fluid mass originally located in the core of the Loop Current Ring and advected by the full, un-filtered model data is indicated in green. The unstable manifolds computed using successively coarser spatial filters are shown by the bold colored lines. The results indicate that the hyperbolic trajectories persist under spatial filtering and that the resulting manifold geometry, at least in the vicinity of strong stretching, is only slightly modified by large changes to the input model velocity. In the language of dynamical systems theory, the invariant manifolds of hyperbolic points are known to be extremely stable structures and this is evident in the vector field of the CUPOM Gulf of Mexico model. The degree to which the details of the manifold geometry persist under perturbation is determined by the magnitude of the hyperbolicity of the generating trajectory. This is also apparent in the figure where the much stronger stretching in the case of *Millennium* results in an LCS boundary that is essentially identical for all levels of smoothing examined. While the error between individual trajectories advected by the filtered and unfiltered fields shows exponential growth in time, the important invariant advective boundaries given by the stable and unstable manifolds are much less sensitive to smoothing.

Results showing synthetic drifter trajectories in the NCOM Adriatic model for similar spatial smoothing operators are shown in Figure 3. The shape of the finite scale Lyapunov exponent (FSLE) as a function of separation distance is shown in figure 4 for both spatially filtered and low pass temporally filtered versions of the raw model data. The FSLE computations consist of initializing a grid of 28,500 initial conditions in the area of interest at nodes of the model velocity field, (dx = dy ~ 1km). At each initial condition, a cluster of 5 particles, arranged in an x-y cross separated by 0.45 km, is advected either forward or backward in time by the model velocity field. In this way, particle separation statistics on each of the 28,500 clusters can be computed and the FSLE found. This quantity, essentially the inverse of the time required to reach a separation scale D, is plotted in figure 4.

Analysis of individual trajectories in the model field indicates extremely strong inertial signals in the interior trajectories with an approximate period of 19 hours. Since inertial contributions are either not captured by, or purposefully removed from Lagrangian trajectory data, we examine what role such time scales play in behavior of the overall development of LCS boundaries and the two particle dispersion as measured by the FSLE. Figure 4 shows FSLE calculations for the raw model output and four different filters, two spatial and two temporal. In all calculations, the scale dependence of the two-particle dispersion is very similar for separation distances larger than ~30 km. Particularly interesting is the strong similarity between the FSLE curves for fields temporally averaged over 36 hours and those spatially filtered with Gaussian window of ~2.5 km. As seen in figure 3, the spatial smoothing does not remove inertial oscillations, while the temporal averaging completely eliminates dynamics on inertial timescales. One conclusion of this is that inertial oscillations play no role in particle pair dispersion.

Results showing maps of the FSLE as functions of initial conditions indicate that the locations of strong intersections of the forward and backward FSLE (indicating hyperbolic trajectories) are preserved under both spatial and temporal averaging. The consequences of this for the construction of manifolds in the smooth fields, and the ability of such smooth field manifolds to organize mixing in the full flow are the subject of current work.

IMPACT/APPLICATIONS

We have demonstrated that information about the geometry of Lagrangian coherent structures given by the location of hyperbolic trajectories and their attending manifolds can be used to generate 'optimal' drifter launch strategies that maximize the environmental information available from necessarily sparse data sets. The results in Molcard et al. (2006) allow application of this directed launch strategy to any Lagrangian data assimilating ocean model in situations where coherent features may identified in the Lagrangian frame.

The results for the GOM model show that the Lagrangian Coherent Structure boundaries, as given by the finite time stable and unstable manifolds of transient hyperbolic points, play an essential role in the kinematics of Loop Current Ring breakup in the Gulf of Mexico. A Lagrangian analysis of several model Loop Current Rings indicates that these structures do not propagate to the western shelf and decay via viscous/bathymetric processes. Instead, the model predicts rapid loss of coherence in deep water around 93W, and thus the break-up of these highly energetic rings is an advective process dominated by the prescence of identifiable hyperbolic features generic in flows with energetic mesoscale eddies. A suite of Lagrangian analysis tools are now in place to rapidly assess model output from other data assimilating GOM models. Plans for analyzing similar dynamics in HICOM Gulf simulations are in place.

The sensitivity results for the CUPOM indicate that the computed LCS boundaries are very robust to perturbations in the underlying Eulerian velocity field. The numerical manifolds, in the vicinity of the transient hyperbolic points, are not effected by small-scale spatial details of the computation and persist under changes which mimic changing eddy-diffusivity values or spatial model resolution. As predicted by the mathematics, (see, for example, Haller and Poje 1999), the stronger the local hyperbolicity in the flow, the more unique and robust are the corresponding finite time numerical manifolds. This is born out in the CUPOM model.

The Lagrangian analysis of a very high resolution NCOM model of the Adriatic has direct bearing on a particular region of importance for planned observational programs. The construction of FSLE maps for the flow off the Gargano peninsula provides information on the size and structure of initial condition sets that are entrained into the reciculation region. Notably, there exist identifiable hyperbolic structures even in the highly time dependent raw model output and these structures persist under averaging and smoothing operations on the velocity field. By examining the effects of temporal and spatial smoothing on the scale-dependent FSLE we have taken the first steps towards constructing a *dispersion* relation for the model output, indicating to what extent the particle pair separation is ergodic.

The shape of the FSLE curves shows no indication of an exponential plateau at smallest resolved scales. If particle dispersion in the Adriatic is indeed a uniformly 'chaotic' process, then the model does not resolve the scales where this occurs. If, and perhaps more likely, the Lagrangian dynamics of the basin are highly non-uniform in space, then there is no single Lyapunov exponent that describes

two particle dispersion. The FSLE curve shows that the statistics of large particle pair separations, such as those produced by isolated, strongly hyperbolic regions are largely independent of the small space and fast time scales in the flow and well represented in the model output.

RELATED PROJECTS

The PI actively collaborates with Professors T. Ozgokmen and A. Griffa at the University of Miami on questions concerning Lagrangian data assimilation, optimal deployment strategies, model-data intercomparison as related to the DART05-06 effort. The PI visited Miami in April 2006 to meet with Ozgokmen and Haza to discuss plans for the Adriatic model analysis. Another collaborative visit is scheduled for this Fall to complete the manuscript on the Adriatic model and to coordinate transfer and access to HICOM Gulf of Mexico archives for future sensitivity and model intercomparison studies.

The PI continues a strong collaborative research program with Professors A.D. Kirwan and B. Lipphardt at the University of Delaware including the concluding studies in a National Science Foundation funded project extending the Lagrangian analyses of the CUPOM Gulf of Mexico model to multiple vertical layers. The PI has participated in the analysis and selection of drifter launch locations based on model output for areas of the South China Sea and has plans for extending the use of SeaWifs based remote sensing data in Lagrangian model-data analyses.

Collaborative work with members of the mathematics faculty at CUNY-CSI and physicists at the University of Massachusetts, Boston on the role of LCS boundaries in the ultimate mixing of diffusive scalar fields will result in the submission of a manuscript to *Phys. Rev. Letters* in the near future.

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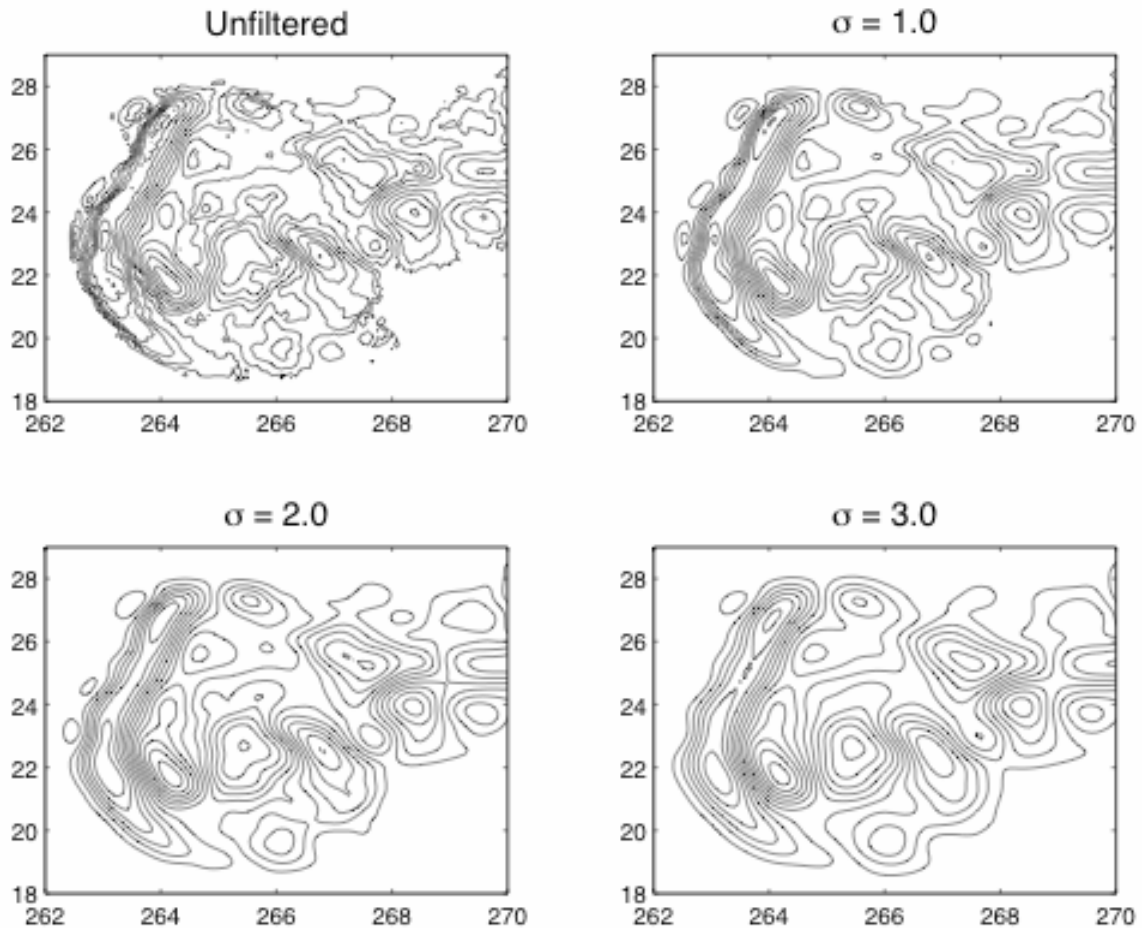
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Figure 1: Effect of spatial filtering of the Gulf of Mexico model output. Shown are contours of the meridional velocity field at a single time for the raw model data and for Gaussian filtering with filter widths of 1, 2, and 3 model grid points (dx ~ 12 km).
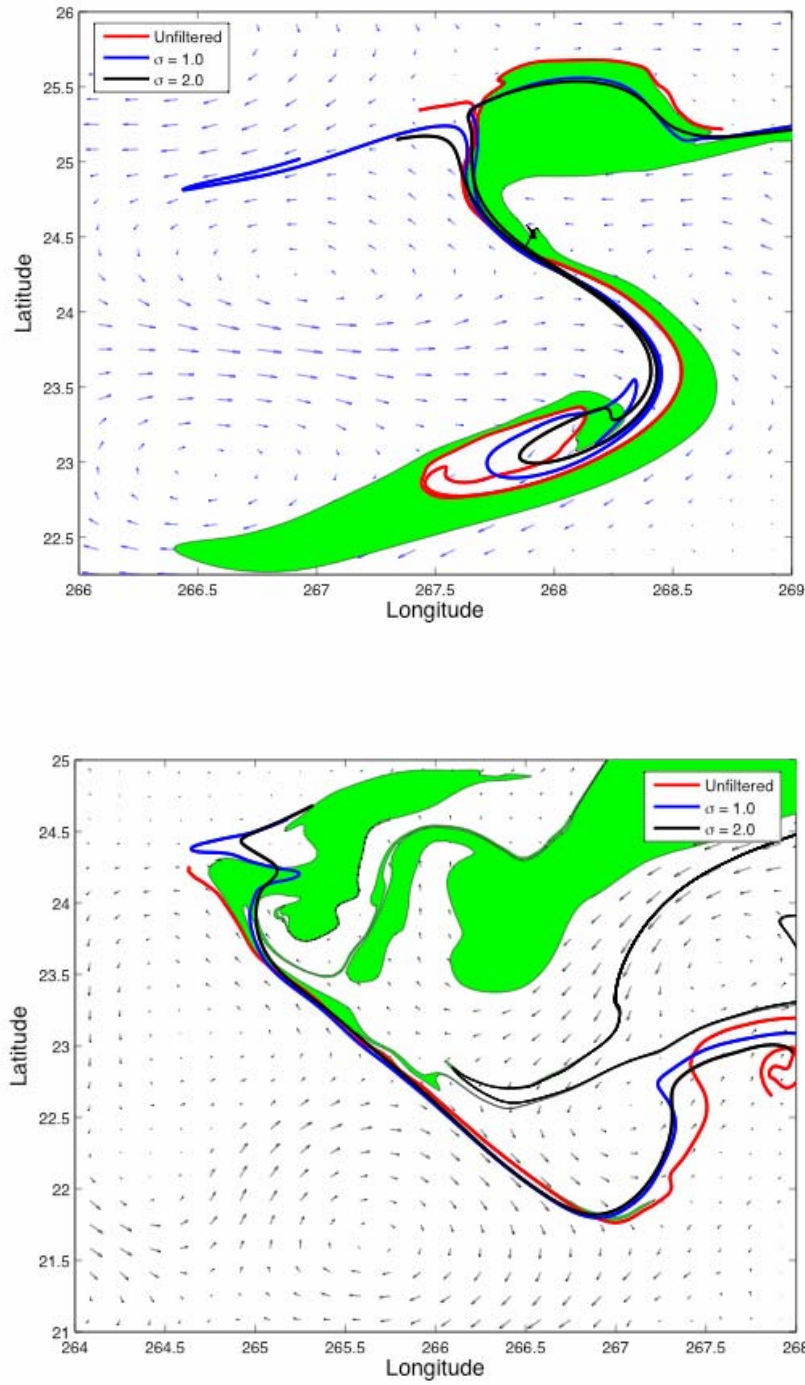
Figure 2: Comparison of the geometry of unstable, out-flowing manifolds in the GOM model constructed from the raw model output and spatially filtered velocity data. The top shows the breakup of the Loop Current Ring Fourchon, the lower figure shows the breakup of eddy Millenium. Note the stability of the structure boundary geometry in the vicinity of the hyperbolic region to coarsening the spatial resolution of the model.
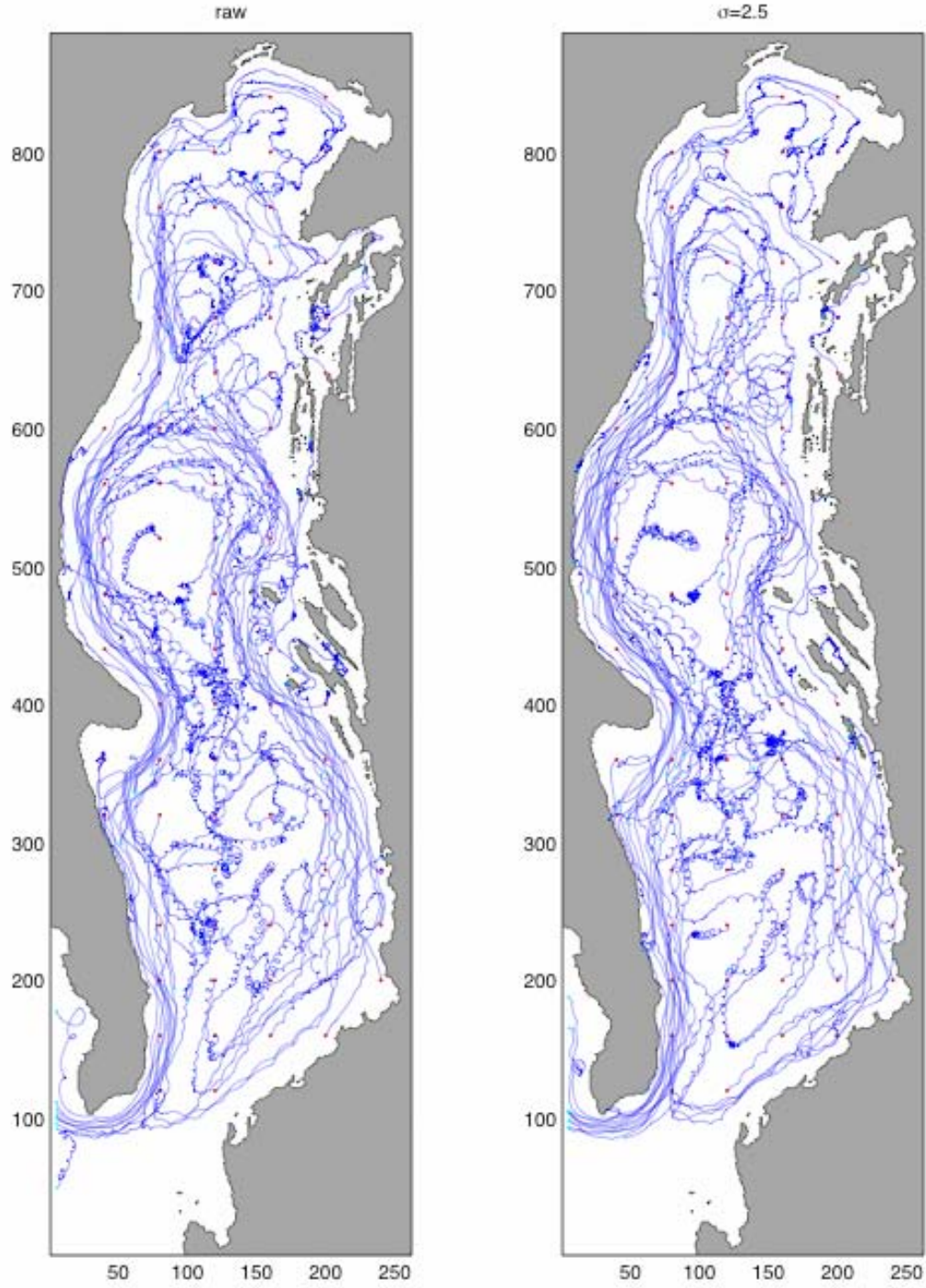
Figure 3: Sample trajectories computed for the raw model output of the NCOM Adriatic model, left, and for the spatially smoothed model velocity with Gaussian filter of width 2.5 grid points (dx ~ 1km).
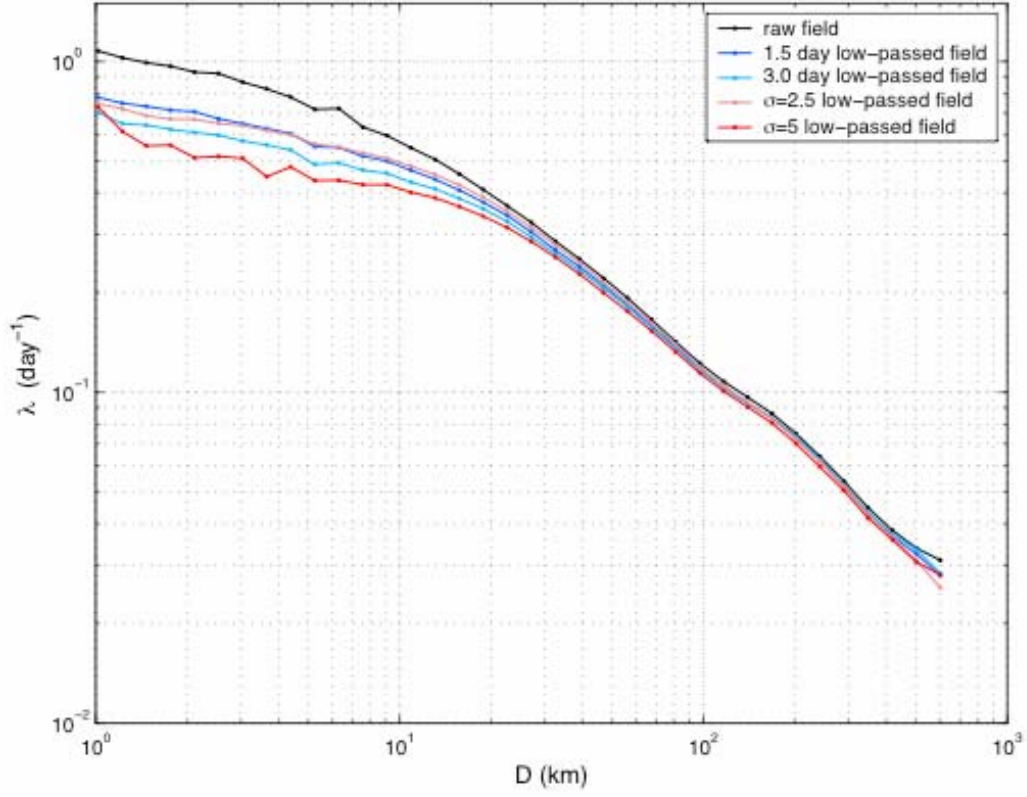
Figure 4: Comparison of the Finite Scale Lyapunov Exponent, measured in 1/day units as a function of separation scale for four different smoothing operators. Note the dispersion times for separation scales larger than 30 km are similar for all cases. Also, the scale-dependent relative dispersion is nearly identical for velocity fields averaged over 1.5 day timescales and instantaneous fields spatially smoothed with Gaussian windows of ~2.5 km.